Coin Identification through Natural Frequency Analysis

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Abstract

The natural frequency of an object is unique to each object like a fingerprint is to a person. Due to hardware limitations, the upper limit of natural frequencies studied was 20 kHz. Each coin was modeled using Computer Aided Design (CAD) software and then natural frequencies were predicted using Finite Element Analysis software. The sound of each coin freely vibrating was then recorded using LabVIEW and processed with the Fast Fourier Transform (FFT) to determine the natural frequencies. Forty coins of each type were measured. All measured coins had natural frequencies within the range of human hearing. The resulting spikes in the FFT graph represented the natural frequency of each coin. This data was then used to create another LabVIEW program that successfully identified which coin was being dropped by recording the sound of the vibration and determining the natural frequency.

Introduction

Powerful singers have been known to sing so loudly and clearly that they break wine glasses. This comes as result of the singer hitting the natural frequency of a wine glass with enough power. Everything has a unique natural frequency. Essentially, the natural frequency is the vibration of an object when it is hit or in some way given mechanical energy. This vibration produces sound waves that can be recorded and measured. The frequency of vibration is dependent on many factors such as mass, shape, density and modulus of elasticity. Fortunately, breaking wine glasses is not the only potential use of this phenomenon.

The natural frequency of an object can be incredibly useful because it always stays the same. An object’s natural frequency can be composed of many different frequency modes. Usually there is one main frequency and then harmonics of that frequency. When these frequencies are identified they can be used as a fingerprint to identify a specific object. These fingerprint frequencies will stay the same regardless of how mechanical energy is given to an object. Coins, just like other objects, have specific measurable natural frequencies. Currently vending machines use electromechanical systems to identify coins. This paper will examine a different approach to identifying individual coins from one another.

This experimental approach involved recording the sound produced by coins freely vibrating and using the frequency domain to identify the different frequencies which are present. The natural frequency of some objects is not easily attained through sound recording. Some materials have internal damping characteristics that prevent the object from vibrating in such a way that its natural frequency can be recorded easily recorded. (e.g. rubber) Generally, stiffer materials have clearer natural frequencies because vibration is more easily propagated through the material. To help get clean ‘fingerprints’ of each coin, a consistent system will be used for measurement. The coins will then be able to be identified by a computer with a microphone.

This study took the principles of natural frequencies and applied them to coins. Once the natural frequency of each coin was numerically modeled and measured, a normal distribution was created of each coin’s natural fre-
frequency spectrum. This data was then used to identify coins of certain value from others. This is an alternative to the mechanical approach currently used by vending machines.

The natural frequency of an object is a useful piece of information. The process that has been used to find the natural frequencies of coins can be used for other objects. This technology has been used to check structural beams for cracking [1,2,3] and could have many other potential engineering uses in the world today. It is important to be able to find the natural frequency of a mechanical system or an object because other systems could have the same natural frequency and could cause interference or damage when they operate together.

**Methods**

The natural frequencies of the coins were first predicted using Finite Element Analysis (FEA) software. Two things are needed to use FEA software; a CAD model and material properties. The Autodesk Inventor CAD system has a built in frequency and modal analysis FEA tool. The coins were modeled as simple disks using dimensions specified by the U.S. Mint [4]. Determining the material properties of the coins was considerably more difficult. A phone call to the U.S. Mint shed some light on why they were so understandably tight lipped about the specific materials used in coins. There is a fear that people will attempt to fake coins for use in vending machines. The only information publicly given is the composition of the coins by mass percentage.

Modern day coins are primarily composed of copper and nickel. The material properties of each coin was estimated by using the publicly available percent composition from the mint and the properties of existing copper nickel alloys, also known as ‘cupronickel’. The modulus of elasticity of the cupronickel was estimated to be 118 GigaPascals (GPa). This is most definitely an estimate because it is not known how the strain hardening of being stamped has effect the modulus of elasticity of the coins. The tensile and yield strength were also estimated from existing alloys.

As shown in Figure 1, FEA systems work by essentially chopping an object into many small pieces that make up a mesh. The more elements that are used for the model, the more accurate the model becomes. The natural frequencies of the coins were estimated using a mesh that contained approximately five thousand elements. The FEA system then modeled the interactions of each piece numerically to give an estimate of each coin’s natural frequency and vibration modes.

The vibrations of a coin after being dropped will be recorded using a standard computer microphone. This will provide raw audio data as shown in Figure 2. Unfortunately, this presents some limitations. These limitations come from the computer hardware that is being employed. A standard sound card is being used. Sound cards employ low-pass filters that block frequencies above 20 kHz to condition audio signals and to prevent aliasing. This limited our natural frequency investigation by microphone to the upper limit of human hearing or 20 kHz. Additionally, microphones have an upper limit on their frequency response between 16 and 20 kHz. This means that frequencies recorded closer to 20 kHz experienced greater attenuation and were harder to discern.

These coins also had additional harmonics that exist above the range of human hearing, but it was not practical to capture them. To do so would have required an expensive microphone with an extraordinarily high frequency response, an external processing device, and extremely high sample rates.

The audio capture sample rate was set to 96,000 samples per second or 96 kHz. This is the maximum sample rate allowed by the computer sound card. The Nyquist theorem tells us that to accurately capture a signal and avoid aliasing we must sample the signal at a rate that is at least twice its frequency.

\[
f_{\text{Nyquist}} = 2f_{\text{Signal}}
\]

Since 20 kHz was the highest possible frequency captured, the audio was sampled at least 4.8 times the highest frequency that was be captured. The optimum sample rate

**Figure 1:** A ‘mesh’ created by the Autodesk Inventor frequency and modal analysis tool. This mesh shows the individual pieces that the coin is cut up into for natural frequency analysis. This particular mesh contains 7,871 elements. Increased mesh size will give more accurate results.
would have been around ten times the highest frequency, but 4.8 was sufficient for natural frequency analysis. If the audio was sampled below 40 kHz it would have been possible for aliasing to create erroneous signals.

Figure 2: Raw time domain data as acquired by LabVIEW. The amplitude of this graph is essentially voltage as detected by the microphone. This raw data contains little valuable information in the time domain. The Fast Fourier Transform was used to retrieve useful data.

Each coin was dropped from the same height and landed the same distance from the microphone each time. To make the interference predictable, a small tile partially composed of quartz has been epoxied to a piece of wood as a landing pad. This is not absolutely necessary since the natural frequency of a coin stays the same even when it may be dropped on different material. Through preliminary analysis it had been shown that the natural frequencies of the quartz, wood, and other common table materials had natural frequency components that were consistently lower than the frequencies of coins. This allowed for the lower end of the frequency spectrum in the frequency domain to simply be thrown out. This low frequency interference region can be easily detected by dropping coins on different surfaces and comparing the resulting FFT graphs.

Figure 3: These peaks represent the natural frequencies of a half dollar coin. They are found from the raw audio data by taking the FFT. The Fourier Transform takes the captured sound data from the time domain and puts it into the frequency domain. In this and other FFT output graphs frequencies below 7 kHz can be disregarded as noise frequencies captured by a dropped coin causing the quartz tile to vibrate. This noise floor is consistent across other surfaces.

A pipe was epoxied to another piece of wood to act as a rail and to ensure consistent drop height. This was mounted to a hinge attached to the same piece of wood that anchors the quartz tile. This system will allow all of the coins to fall into the same place from the same height with consistent kinetic energy.

Coins of the same monetary value were found to have some variance in their natural frequencies. To account for this, forty random coins of each denomination had their natural frequencies measured. The data from the drops were compiled to produce a normal distribution of each coin’s natural frequency. The sample standard deviation for each coin was also computed. This allows the experiment the ability to claim to be able to recognize 95% of coins dropped. This confidence interval represents all values within two standard deviations from the sample mean.

LabVIEW was used to record the sound data. Each time sound data was acquired, LabVIEW computed the Fast Fourier Transform (FFT) of the data.

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\[ X_k = \sum_{i=0}^{n-1} x_n \cdot e^{-j2\pi ik/n} \] (2)

The FFT took the sound data out of the time domain (x axis is time) and into the frequency domain (x axis is Hz). Attempting to recognize unique frequencies in the time domain would simply not be possible. The Fourier Transform is what made this experiment possible because it showed what frequencies were recorded and how often they occurred relative to one another. The sound data that was processed was composed of hundreds of thousands of discrete samples. Therefore, the Discrete Fourier Transform is the specific variation of the Fourier Transform that will be used. Through the use of this equation and some logic programming, LabVIEW was able to identify individual coins when certain amplitudes were reached in specific frequency ranges. Several of the coin natural frequencies overlapped each other. This potentially would have been a problem for the LabVIEW program, but the use of boolean algebra made it possible to ensure that only the correct coin is identified since the coins have different natural frequency harmonics.

Results and Discussion

Penny

According the U.S. Mint the modern day penny is composed of 2.5% copper and 97.5% zinc. A penny is essentially zinc with copper plating. The material properties used for the FEA software were estimated from the properties of zinc. This was used to then calculate the natural frequency modes of the penny. Surprising the first natural frequency mode was found to exist at approximately 13.1 kHz, a frequency well within human hearing. The first mode of all the coins is the ‘taco’ shape. If a high speed camera with a high enough frame rate was used, it would be possible to see the actual deflection of the coin in that shape. Figure 5 represents an exaggeration of the coin deformation.

No usable sound data was captured that revealed the natural frequency of the penny. There are a number of possible causes for this. It is possible that because of the composition of the penny there is a significant amount of damping. It is also possible that the penny absorbs a significant amount of energy as it bounces which prevents a significant amount of free vibration which produces the sound waves that are picked up by the microphone. The penny probably just isn’t large enough to produce sound waves of high enough amplitude. Initially it was thought that it was possible that the natural frequency of a penny was higher than the range of human hearing, but the numerical model suggests otherwise.

Nickel

To create a numerical model of the natural frequency modes of the nickel, a copper alloy of 25% nickel 75% was used per U.S. Mint specifications. The modulus of this material was approximated at 118 GPa. The FEA software estimated that the first two modes natural frequency would land around 17.3 kHz. The third mode of natural frequency was calculated to be approximately 29.2 kHz.

When the numbers from the numerical model are compared with the measured values it becomes apparent that there is a considerable amount of error. There are a number of possible explanations for this erroneous behavior. The first is that the material used in the FEA could actually be quite far from what the actual properties of a nickel are. The material properties were estimated from known copper alloys, but when the coin was stamped at the mint it is possible that it could have undergone a significant amount of strain hardening which would affect its modulus of elasticity significantly even though it is the same material. Another possible source of error comes from the size and shape of the coin. The CAD model was designed from diameter and thickness dimensions given by the U.S. Mint, but that did not take into account the detailed features that exist on the face of coins. The research showed that the natural frequency of a coin could vary by several hundred Hz simply from the edges of a coin being worn away.
Figure 4: This is the result of taking the Fourier transform of raw audio capture data from a dropping nickel. It can be seen that this particular nickel has one natural frequency mode within the range of human hearing at about 16 kHz. There was a considerable amount of variation between different years of nickels.

Unlike the penny, sound data from the nickel was successfully captured. As shown in Figure 4, the natural frequencies of forty nickels were measured. A sample mean and sample standard deviation were calculated. The sample mean was found to be around 12.7 kHz for the first two natural frequency modes and 14.4 kHz for the third frequency. The first two modes of natural frequency are generally very close together for coins, so much so that they can become indistinguishable in the FFT due to its resolution. For the purpose of this experiment the first two modes of natural frequency are generally averaged together. The standard deviation was 797 Hz and 617 Hz respectively.

Twenty-nine nickels were used to test the accuracy of the coin identifying program. All twenty-nine coins were successfully identified by LabVIEW as nickels. This improves upon the 95% positive identification as was theorized.

Dime

According to the U.S. Mint, dimes are composed of Nickel and Copper. The same material was used as was used to estimate the nickel’s natural frequencies. The numerical model predicted the first two modes of natural frequency to be approximately 15.2 kHz. This is a few kHz above the frequencies that were measured. The next mode of natural frequency was found to be around 26 kHz, well outside the range of human hearing. As seen in Figure 6, the modal deformation looks much like a drum. Using the same material as the nickel definitely added error to the model since the nickel and dime have different weight compositions of copper and nickel. However, the mode of vibration is accurate. It shows that, unlike the first two modes, the third mode of natural frequency comes from the center of the coin. It looks like a drum moving up and down.

Figure 6: This figure depicts the third mode of the natural frequency of the dime. The computed frequency from FEA is approximately 26 kHz. This mode of vibration is through the deformation much like beating a drum. The first two modes are identical to the first two modes of other disk shaped objects.

Capturing good data from the dime was somewhat difficult. If the dime didn’t land close the microphone it usually didn’t pick up the vibrations. The copper nickel alloy that composes coins other than pennies has very good vibrating qualities. Just as with the other coins, a sample size of 40 was used to determine the sample mean and sample standard deviations.

Two natural frequency modes were found within 20 kHz. As seen in Figure 7, they were found to be at 12.3 kHz and 12.8 kHz. The sample standard deviations are around 250 Hz for each frequency. This is a great improvement over the nickel. It is possible that since the nickel has undergone several redesigns over the years that much of the variation could be blamed on this redesign. The dime is a much tighter frequency range.

Figure 5: This image shows the estimated first mode of natural frequency of a nickel. It has the same shape as the penny does because they are both disk shaped objects. The deflection is somewhat exaggerated to clearly show the deformation of the coin.

Figure 7: This figure depicts the third mode of the natural frequency of the dime. The computed frequency from FEA is approximately 26 kHz. This mode of vibration is through the deformation much like beating a drum. The first two modes are identical to the first two modes of other disk shaped objects.
Despite being smaller than the penny, the dime has two natural frequency peaks within the range of human hearing. These peaks are fairly consistent across other types of dimes because of little redesigning done to dimes over the years.

Once LabVIEW was calibrated to the frequency ranges of the dime, forty-one dimes were measured. Initially there was some difficulty in getting a reliable result that was not confused with the nickel. The nickel and dime slightly overlap in their ranges. After some calibration 97.56% of the dime were identified correctly. This value is better than the predicted 95% confidence interval.

Quarter

The quarter was modeled using the same material as the dime and the nickel. The FEA software estimated its first three natural frequency modes to exist at 10.807 kHz, 10.808 kHz, and 18.2 kHz with the same modal shapes as the other coins. The predicted first two modes of natural frequency were actually close to the measured values.

As seen in Figure 8, after measuring the natural frequencies of forty quarters the sample mean natural frequencies were found to be at 9.1 kHz, 9.3 kHz, and 15.4 kHz. The standard deviation of each frequency was around 250 Hz. This is a similar standard deviation of the dime. Unlike the nickel, the changes made to the quarter in the last twenty-five years have not had a significant impact on its natural frequency.

Half Dollar

The half dollar was modeled in the same manner as the other coins using the same material that the quarter, dime, and nickel used. The FEA software predicted natural frequency modes to exist at 8.3 kHz, 14.2 kHz, and 18.8 kHz. As shown in Figure 9, the fourth mode of natural frequency is a more complicated combination of the first and second modes.

The half dollar is by far the largest coin in circulation. Acquiring forty of them for statistical analysis was not a simple feat. As seen in Figure 3, the measured natural frequencies were at 7.3 kHz, 13.3 kHz, and 16.5 kHz. These values are similar to the values predicted by the FEA software. The standard deviation for the half dollar frequencies was around 200 Hz.

The LabVIEW program was able to successfully identify 100% of the forty-three half dollars dropped. The larger coins have been easier to identify because they have at
least three distinct frequencies within the range of human hearing across a broad spectrum.

Figure 9: This is the fourth natural frequency mode of the half dollar. The FEA software estimates that this mode occurs around 18 kHz.

Dollar Coin

The dollar coin was modeled as a disk but a soft yellow brass was used to represent its composition. It is actually composed of brass and manganese. Since the specific type of alloy used by the mint to produce these coins couldn’t be located, the effect of the manganese on the modulus of elasticity was left out. The FEA software predicted natural frequencies at 9.9 kHz, 9.9 kHz, and 17.1 kHz. It was shown to have the same modes of natural frequency as the other coins.

Forty dollar coins had their natural frequencies measured. The frequencies were found to be at 8.4 kHz, 8.6 kHz, and 14.4 kHz. The dollars coins are actually not a very popular coin. Because of this, their sample standard deviations were very tight. The average standard deviation was less that a hundred hertz. This made the coins very easy to specifically identify. LabVIEW was able to correctly determine dollar coins one hundred percent of the time. The coins have had very little circulation and very little wear.

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References


